RN-003-001513 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

February – 2019

Mathematics : BSMT-501(A)

(Mathematical Analysis-1 & Group Theory)

(Old Course)

Faculty Code : 003
Subject Code : 001513

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) **All** questions are compulsory.

(2) Figure to the **right** indicate full marks of the question.

1 Answer the following questions : **20**

(1) Define : Interior point.

(2) Define : Discrete metric space.

(3) In usual notation define $U(P, f)$.

(4) Define : Norm of the partition.

(5) Define : Group.

(6) Define : Group Homomorphism.

(7) State Lagrange's Theorem for group.

(8) True or False : Every bounded function on $[a, b]$ is R-integrable.

(9) True or False : $(\mathbb{Z}, -)$ is a group.

(10) Let G be a group with identity element e and let $x, y, x, w \in G$.

If $x^2 y^{-1} z^5 w = e$, they y = ___________.

(11) Let G be a group and $a, b, c, d \in G$. Then $(abcd)^{-1} =$ _______.

(12) If $(\mathbb{R}, d)$ is a discrete metric space, then find $N(\pi, 1/2)$.

(13) If $(\mathbb{R}, d)$ is a usual metric space, then find (0, 3)'.

(14) Let $A = \left\{ \frac{1}{2n} \mid n \in \mathbb{N} \right\} \subset \mathbb{R}$. Find $\bar{A}$.

(15) Let $f(x) = x, x \in [0,1]$ and $P = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$ be a partition of $[0,1]$. Compute $U(P,f)$.

(16) Let $*$ be defined as $a * b = \frac{ab}{10}, a, b \in \mathbb{Q}$. What is the identity for $*$?

(17) Express $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 8 & 1 & 7 & 4 & 9 & 5 & 3 & 6 \end{pmatrix} \in S_9$ as a product of disjoint cycles.

(18) Let G be a group with identity e and $b \in G$. If $b^4 = e$, then list all possible orders of element b.

(19) Let $*$ be defined as $a * b = \frac{ab}{15}, a, b \in \mathbb{Q}$. What is the inverse of 3 ?

(20) List all invertible elements of $(\mathbb{Z}_9, \times_9)$.

2 (A) Attempt any **THREE** : **6**

(1) Show that $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}, d(x,y) = | x^2 - y^2 |$ is not a metric on $\mathbb{R}$.

(2) Show that $\mathbb{N}$ is closed subset of $\mathbb{R}$.

(3) By an example show that : arbitrary union of closed sets may not be closed.

(4) State First Mean Value Theorem of Integral Calculus.

(5) If $f : [a,b] \to \mathbb{R}$ is a bounded and $P \in P[a,b]$, then show that $L(P,f) \leq U(P,f)$.

(6) If $f : [a,b] \to \mathbb{R}$ is a bounded and $P \in P[a,b]$, then show that $U(P,-f) = -L(P,f)$.

(B) Attempt any **THREE** : **9**

(1) Prove that if (X,d) is a metric space, then

$| d(x,z) - d(y,z) | \leq d(x,y), \forall x, y, z \in X$.

(2) Prove that : Any finite subset of a metric space is closed.

(3) Define Cantor Set.

(4) If $f \in R[a,b]$, then show that

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a),$$

where m and M are the infimum and supermum of f on [a,b,], respectively.

(5) If $P_1, P_2 \in P[a,b]$, then show that $L(P_1, f) \le U(P_2, f)$.

(6) Using second definition prove that $\int_1^2 2x\,dx = 3$.

(C) Attempt any **TWO** : **10**

(1) If (X, d) is a metric space, then show that

$$d_1 : X \times X \to \mathbb{R};\quad d_1(x,y) = \frac{d(x,y)}{1+d(x,y)} \text{ is}$$

a bounded metric on X.

(2) Lex X be a metric space and $A, B \subset X$. Show that $(A \cap B)^\circ = A^\circ \cap B^\circ$.

(3) Using second definition prove that

$$\int_0^1 (2x^2 - 3x + 5)\,dx = \frac{25}{6}.$$

(4) State and prove Darboux's Theorem.

(5) Show that :

$$\lim_{n\to\infty}\left[\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n}\right] = \log_e 2.$$

3 (A) Attempt any **THREE** : **6**

(1) Let G be a finite group and $a \in G$. Prove that $O(a)$ divides $O(G)$.

(2) Let G be a group and $a, b \in G$. Show that $(ab)^{-1} = b^{-1}a^{-1}$.

(3) Check whether $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 7 & 4 & 6 & 5 \end{pmatrix} \in S_7$ is odd or even ?

(4) Give two reasons why the set odd integers is not a group under usual addition ?

(5) By an example show that the union of two subgroups of a group may not be a subgroup of the group.

(6) Find order of $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 1 & 7 & 8 & 4 & 6 & 5 \end{pmatrix} \in S_8$.

(B) Attempt any **THREE** : **9**

(1) Find the remainder obtained on dividing 3^{256} by 14.

(2) Prove that a group of prime order is cyclic.

(3) Let G be a group. If $a^2 = e, \forall a \in G,$ then show that G is abelian.

(4) Let G be a group and $a \in G$. Show that $O\left(a^{-1}\right) = O(a)$.

(5) Let G be a group and $H_1, H_2 \leq G$. Show that $H_1 \cap H_2 \leq G$.

(6) In a group show that the identity element is unique.

(C) Attempt any **TWO** : **10**

(1) State and prove Cayley's Theorem.

(2) Show that the set $G = \{f_{a,b} \mid f_{a,b} : \mathbb{R} \to \mathbb{R}, f_{a,b}(x) = ax + b\,(x \in \mathbb{R}), a \neq 0, a, b \in \mathbb{R}\}$ is a group under composition of mappings.

(3) State and prove Lagrange's Theorem.

(4) Let G be a group. For $n \in \mathbb{Z}$ prove that $(aba^{-1})^n = ab^n a^{-1}\ \forall a, b \in G.$

(5) Prove that : For $n \geq 3,$ every $f \in A_n$ can be express as product of 3-cycle.
